

Single electron control in n-type semiconductor quantum dots using non-Abelian holonomies generated by spin orbit coupling

S.-R. Eric Yang^{1,2*} and N.Y. Hwang¹

¹*Physics Department, Korea University, Seoul Korea*

²*School of Physics, Korea Institute for Advanced Study, Seoul Korea*

We propose that n-type semiconductor quantum dots with the Rashba and Dresselhaus spin orbit interactions may be used for single electron manipulation through adiabatic transformations between degenerate states. All the energy levels are discrete in quantum dots and possess a double degeneracy due to time reversal symmetry in the presence of the Rashba and/or Dresselhaus spin orbit coupling terms. We find that the presence of double degeneracy does not necessarily give rise to a finite non-Abelian (matrix) Berry phase. We show that a distorted two-dimensional harmonic potential may give rise to non-Abelian Berry phases. The presence of the non-Abelian Berry phase may be tested experimentally by measuring the optical dipole transitions.

PACS numbers: 71.55.Eq, 71.70.Ej, 03.67.Lx, 03.67.Pp

I. INTRODUCTION

Single electron control in semiconductor quantum dots would be valuable for spintronics, quantum information, and spin qubits[1]. Adiabatic time evolution of degenerate eigenstates of a quantum system provides a means for controlling individual quantum states[2, 3, 4]. They exhibit non-Abelian gauge structures and often give finite non-Abelian Berry phases (they are also called matrix Berry phase or holonomic phase). These phases often depend on the geometry of the path traversed in the parameter space of the Hamiltonian. Nuclei[5], superconducting nanocircuits[6], optical systems[7], and atomic systems[8, 9, 10] have such degenerate quantum states. It has been shown that universal quantum computation is possible by means of non-Abelian unitary operations[11, 12]. Manipulation is expected to be stable since symmetries of the Hamiltonian that give rise to degeneracy are not broken during the adiabatic transformations. However, the degree of its stability is under investigation[13].

Recently several interesting possibilities for electronic manipulation in semiconductors have been proposed. Spin manipulation of quasi-two-dimensional electrons by time-dependent gate voltage is possible through the Dresselhaus and Rashba spin-orbit coupling mechanisms [14]. A geometric spin manipulation technique based on acceptor states in p-type semiconductors with spin-orbit coupling was proposed[15]. Spin-orbit coupling and a revolving external electric field may generate holonomic qubit operations in CdSe nanocrystals [16]. Holonomic quantum computation using excitons in semiconductor nanostructures has been also proposed[17].

A matrix Berry phase is experimentally interesting because it represents mixing between degenerate levels. Let us explain briefly the basic ideas using a sim-

ple system possessing Kramers' double degeneracy[8]. The Hamiltonian depends on some external parameters λ_p . If the system is in a superposition state $|\psi'(0)\rangle = c_1(0)|\psi_1(0)\rangle + c_2(0)|\psi_2(0)\rangle$ at time $t=0$ an adiabatic evolution of the parameters λ_p can transform this state into another state $|\psi'(t)\rangle = c_1(t)|\psi_1(t)\rangle + c_2(t)|\psi_2(t)\rangle$ after some time t . Here the orthonormal basis states $|\psi_i(t)\rangle$ are the instantaneous eigenstates of the Hamiltonian. For a cyclic change with the period T , represented by a closed contour C in the parameter space, the states $|\psi_1(T)\rangle$ and $|\psi_2(T)\rangle$ return to the initial states $|\psi_1(0)\rangle$ and $|\psi_2(0)\rangle$, but the coefficients $c_1(T)$ and $c_2(T)$ may not return the initial values. In such a case a 2×2 matrix Berry phase (non-Abelian Berry phase) Φ_C is generated

$$\begin{pmatrix} c_1(T) \\ c_2(T) \end{pmatrix} = \Phi_C \begin{pmatrix} c_1(0) \\ c_2(0) \end{pmatrix}. \quad (1)$$

This non-Abelian geometric phase (holonomy) is connected to non-Abelian gauge potentials, as we discuss below. During the adiabatic cycle the degenerate energy $E(t)$ varies with time. But hereafter we will set $E(t) = 0$ since it can be easily restored when needed. The expansion coefficients satisfy the time-dependent Schrödinger equation

$$i\hbar\dot{c}_i = - \sum_j A_{ij}c_j \quad i = 1, 2. \quad (2)$$

The matrix elements A_{ij} are given by $A_{ij} = \hbar \sum_p (A_p)_{i,j} \frac{d\lambda_p}{dt}$, where the sum over p in A_{ij} is meant to be the sum over λ_p . A_p are 2×2 matrices and are called the non-Abelian vector potentials. They are given by

$$(A_p)_{i,j} = i\langle\psi_i|\frac{\partial\psi_j}{\partial\lambda_p}\rangle. \quad (3)$$

The formal solution of the time-dependent Schrödinger equation gives the matrix Berry phase $\Phi_C = P e^{-\oint_C \sum_p A_p d\lambda_p}$, where P represents a path ordering.

*eyang@venus.korea.ac.kr

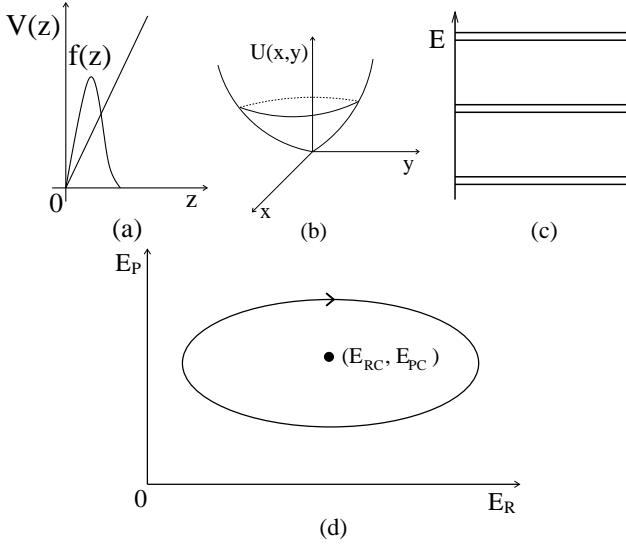


FIG. 1: (a) An electric field along the z -axis quantizes the electronic motion in a triangular potential along the axis. In our work the triangular potential is sufficiently strong and only the lowest energy subband is included. The structural inversion asymmetry in $V(z)$ leads to the Rashba interaction. An adiabatic change can be induced by changing the magnitude of the electric field. (b) The two-dimensional electronic motion is quantized in a distorted harmonic potentials $U(x, y) = \frac{1}{2}m^*\omega_x^2x^2 + \frac{1}{2}m^*\omega_y^2y^2 + \epsilon'y$. An adiabatic change can be induced by changing the magnitude of the distortion potential. (c) In the presence of the Rashba and/or Dresselhaus spin orbit coupling terms each discrete eigenstate of a semiconductor quantum dot has a double degeneracy due to time reversal symmetry in the absence of a magnetic field. The three lowest energy levels are shown schematically (Each pair is doubly degenerate). (d) A schematic drawing of a cyclic adiabatic path. The parameters E_R and E_P depend on the magnitudes of the electric field and the deviation from the two-dimensional harmonic potentials.

Under a unitary transformation $|\psi'_i\rangle = \sum_j U_{ij}^* |\psi_j\rangle$ the non-Abelian gauge structure emerges

$$A'_k = U A_k U^\dagger + iU \frac{\partial U^\dagger}{\partial \lambda_k}. \quad (4)$$

Here we propose that quantum dots in n -type semiconductors with spin-orbit interaction can have matrix Berry phases. All the discrete energy levels of quantum dots possess a double degeneracy because the Rashba and/or Dresselhaus spin orbit coupling terms have time reversal symmetry. In II-VI semiconductors the Rashba term is expected to be larger than the the Dresselhaus coupling. In III-V semiconductors, such as GaAs, the opposite is true[18]. In our work both the Rashba and Dresselhaus terms are included. The adiabatic transformation can be performed electrically by changing the confinement potentials of the quantum dot, see Fig.1(d). We find that, although the spin orbit terms formally give rise to a non-Abelian structure for the matrix vector potentials,

double degeneracy does not necessarily lead to finite non-Abelian Berry phases. We show that when the circular symmetry of two-dimensional harmonic potentials is broken matrix Berry phases can be produced (such a distortion is shown in Fig.1(b)). We propose that the presence of a matrix Berry phase may be detected by measuring the the optical spectrum.

In Sec. II we describe the Hamiltonian of the system in detail, and in Sec. III we discuss a model calculation of the matrix Berry phase. A possible experimental detection of the the matrix Berry phase is suggested in Sec. IV. Conclusions are given in Sec. V.

II. HAMILTONIAN

An electric field E is applied along the z -axis and electrons are confined in a triangular potential $V(z)$, see Fig.1(a). When the the width of the quantum well along the z -axis is sufficiently *small* we may include only the lowest subband state along the z -axis. We denote this wavefunction by $f(z)$, see Fig.1(a). From the expectation value $\langle f(z) | k_z^2 | f(z) \rangle = 0.8(2m^*eE/\hbar^2)^{2/3}$ we estimate the characteristic length scale along the z -axis: $R_z = 1/\sqrt{0.8(2m^*eE/\hbar^2)^{2/3}}$. The Hamiltonian in the absence of the spin orbit coupling is $H_K = -\frac{\hbar^2 \nabla^2}{2m^*} + U(x, y) + V(z)$. We take the two-dimensional potential to be $U(x, y) = \frac{1}{2}m^*\omega_x^2x^2 + \frac{1}{2}m^*\omega_y^2y^2 + V_p(x, y)$, see Fig.1(b). The strengths of the harmonic potentials are denoted by ω_x and ω_y . They may be varied by *changing gate potentials* of the quantum dot system. The characteristic lengths scales along x - and y -axis are $R_{x,y} = \sqrt{\frac{\hbar}{m^*\omega_{x,y}}}$. The potential $V_p(x, y) = \epsilon'y$ represents a distortion of the harmonic potentials and its strength ϵ' may be *varied electrically*.

In a periodic crystal potential of a semiconductor the spin orbit interaction has two contributions. The Rashba spin orbit term is

$$H_R = c_R (\sigma_x k_y - \sigma_y k_x). \quad (5)$$

Here $\sigma_{x,y}$ are Pauli spin matrices and $k_{x,y}$ are momentum operators ($k_x = \frac{1}{i} \frac{d}{dx}$ and similarly with k_y). The constant c_R depends on the external electric field E applied along the z -axis. The Dresselhaus spin orbit term is

$$H_D = c_D ((\sigma_x k_x (k_y^2 - k_z^2)) + (\sigma_y k_y (k_z^2 - k_x^2))). \quad (6)$$

There is another term of the form $\sigma_z \langle k_z \rangle (k_x^2 - k_y^2)$ in the Dresselhaus spin orbit term but it vanishes since the expectation value $\langle k_z \rangle = \langle f(z) | k_z | f(z) \rangle = 0$ for the first subband along z -axis. The constant c_D represents breaking of inversion symmetry by the crystal in zinc blende structures.

The total Hamiltonian of an electron in a semiconductor quantum dot is $H = H_K + H_R + H_D$. An eigenstate

of the Hamiltonian H may be expanded as a linear combination of the eigenstates of H_K

$$|\psi\rangle = \sum_{mn} c_{mn\uparrow} |mn \uparrow\rangle + \sum_{mn} c_{mn\downarrow} |mn \downarrow\rangle. \quad (7)$$

The expansion coefficients satisfy a matrix equation $\sum_{m'n'\sigma'} \langle mn\sigma | H | m'n'\sigma' \rangle c_{m'n'\sigma'} = E c_{mn\sigma}$. In the basis states $|mn\sigma\rangle$ the quantum number $m(n)$ and σ label the harmonic oscillator levels along the x-axis (y-axis) and the component of electron spin. The subband wavefunction $f(z)$ is suppressed in the notation $|mn\sigma\rangle$.

In the absence of the Zeeman term the total Hamiltonian is invariant under time reversal symmetry: $\vec{k} \rightarrow -\vec{k}$ and $\vec{\sigma} \rightarrow -\vec{\sigma}$. The time reversal operator is $K = -i\sigma_y C$, where the operator C stands for complex conjugation. The time reversed state of $|\psi\rangle$ is

$$|\bar{\psi}\rangle = K|\psi\rangle = -\sum_{mn} c_{mn\downarrow}^* |mn \uparrow\rangle + \sum_{mn} c_{mn\uparrow}^* |mn \downarrow\rangle. \quad (8)$$

Note that $K^2|\psi\rangle = -|\psi\rangle$. These two states are degenerate and orthonormal. We have suppressed the Bloch wavefunction of the conduction band in applying the time reversal operator since it is unaffected by the operator K . Our wavefunctions are all effective mass wavefunctions and only the conduction band Bloch wavefunction at $\vec{k} = 0$ is relevant.

An adiabatic change implemented by changing the energy parameters E_R and E_p that characterize the Rashba term and the distortion potential $V_p(x, y)$:

$$E_R = \frac{c_R}{\sqrt{2}R_y} \text{ and } E_p = \epsilon R_y, \quad (9)$$

where $\epsilon = \epsilon'/\sqrt{2}$. The first parameter E_R depends on the electric field through the constant c_R . The typical value of the energy scale associated with the Rashba constant depends on the electric field applied along the z-axis and the semiconductor material: it is of order $E_R = c_R/R \sim 0.01 - 10 \text{ meV}$, where the length scale $R \sim 100 \text{ \AA}$ is the lateral dimension of the quantum dot. The second adiabatic parameter E_p represents the strength of the distortion potential $\epsilon'y$: the expectation value of the distortion potential is $E_p = \langle 0 | \epsilon'y | 1 \rangle$. Its magnitude is of order $1 - 10 \text{ meV}$, depending on the electric field applied along the y-axis and the width of the triangular potential $V(z)$.

III. MODEL CALCULATION OF NON-ABELIAN BERRY PHASE

A. A truncated Hamiltonian matrix

We work out a model that can be solved analytically. This model is simple but much can be learned from it. Let us take $\omega_x = 2\omega_y$ (Other values of ω_x can also be chosen). Then the lowest eigenenergy state of H_K is

$|mn\rangle = |00\rangle$ with the energy $E_0 = \frac{3}{2}\hbar\omega_y$ and the next lowest eigenenergy state is $|01\rangle$ with the energy $E_1 = \frac{5}{3}E_0$. The typical value of the energy spacing between the quantum dot levels, E_0 , is of order $1 - 10 \text{ meV}$. Let us write down the eigenstates of the total Hamiltonian as a linear combination of four basis states made out of these states and spin degree of freedom:

$$|\psi\rangle = c_{0,0,\uparrow} |0, 0, \uparrow\rangle + c_{0,1,\uparrow} |0, 1, \uparrow\rangle + c_{0,0,\downarrow} |0, 0, \downarrow\rangle + c_{0,1,\downarrow} |0, 1, \downarrow\rangle. \quad (10)$$

The 4×4 truncated Hamiltonian matrix is

$$\begin{pmatrix} E_0 & E_p & 0 & -iE_R - E_D \\ E_p & E_1 & iE_R + E_D & 0 \\ 0 & -iE_R + E_D & E_0 & E_p \\ iE_R - E_D & 0 & E_p & E_1 \end{pmatrix}, \quad (11)$$

or

$$\begin{aligned} & \frac{1}{2}(E_0 + E_1)I + \frac{1}{2}(E_0 - E_1) \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix} \\ & + E_p \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_x \end{pmatrix} + E_R \begin{pmatrix} 0 & \sigma_y \\ \sigma_y & 0 \end{pmatrix} \\ & + E_D \begin{pmatrix} 0 & -i\sigma_y \\ i\sigma_y & 0 \end{pmatrix}. \end{aligned} \quad (12)$$

The matrix elements of the distortion potential are $\langle m | y | n \rangle = \sqrt{\frac{\hbar}{2m^*\omega_y}}(\sqrt{n+1}\delta_{m,n+1} + \sqrt{n}\delta_{m,n-1})$. Note that the distortion potential couples even and odd parity states of the one-dimensional harmonic potential. Other functional form of $V_p(x, y)$ may be also used to generate the non-Abelian Berry phase as long as it couples even and odd parity states. The first term in the Dresselhaus spin orbit term, Eq. (6), is zero since $\langle 0 | k_x | 0 \rangle = 0$. From the second term of the Dresselhaus term we get $-ic_D \langle 0 | k_y | 1 \rangle (\langle f(z) | k_z^2 | f(z) \rangle - \langle 0 | k_z^2 | 0 \rangle) = -E_D$, where the constant $E_D = f(E_R) - g(E_0)$ with $f(E_R) = c_D/R_y R_z^2$ and $g(E_0) = c_D/R_y R_x^2$. Here we have used the momentum matrix elements $\langle m | \hbar k_{x,y} | n \rangle = i\sqrt{\frac{m^*\hbar\omega_{x,y}}{2}}(\sqrt{n+1}\delta_{m,n+1} - \sqrt{n}\delta_{m,n-1})$. The magnitude of the energy scale associated with the Dresselhaus term is of order $E_D = c_D/R^3$, and it can be larger or smaller than the Rashba term, depending on the material[18]. The function $f(E_R)$ depends on E_R because the electric field E enters through R_z . There is no simple way to calculate c_R because it depends both on the electric field inside the semiconductor heterostructure and on the detailed boundary conditions at the interface. For simplicity we take $f(E_R) = aE_R$ and $g(E_0) = bE_0$, where a and b are numerical constants. More complicated functions, for example, $f(E_R) = E_R^2/E_0$ and $g(E_0) = E_0$, could be used, but our calculation indicates that the essential physics does not change.

B. Finite matrix Berry phase

Diagonalization of this 4×4 Hamiltonian matrix gives the eigenenergies $\lambda = \frac{1}{3}(4E_0 \pm \sqrt{E_0^2 + 9E_D^2 + 9E_p^2 + 9E_R^2})$. Let us choose the following state

$$|\psi\rangle = \frac{1}{N} \begin{pmatrix} 3E_p \\ E_0 - \sqrt{E_0^2 + 9(E_D^2 + E_p^2 + E_R^2)} \\ 3(E_D - iE_R) \\ 0 \end{pmatrix}, \quad (13)$$

and its time reversal state

$$|\bar{\psi}\rangle = \frac{1}{N^*} \begin{pmatrix} -3(E_D + iE_R) \\ 0 \\ 3E_p \\ E_0 - \sqrt{E_0^2 + 9(E_D^2 + E_p^2 + E_R^2)} \end{pmatrix}. \quad (14)$$

as the basis states in the lowest energy degenerate Hilbert space.

In the evaluation of the matrix A_p we use $2 \int \phi_k(r)^* \frac{\partial \phi_k(r)}{\partial \lambda_p} = \frac{\partial}{\partial \lambda_p} \int |\phi_k(r)|^2 dr = 0$, where $k = mn\sigma$. Note that $\phi_k(r)$ is the two-dimensional harmonic oscillator wavefunction and that it is a real function. One can show that $(A_p)_{i,j} = i \sum_k \alpha_k^* \frac{\partial \beta_k}{\partial \lambda_p}$, where the pair of degenerate states are $|\psi_i\rangle = \sum_k \alpha_k |k\rangle$ and $|\psi_j\rangle = \sum_k \beta_k |k\rangle$. The orthonormalization $\langle \psi_i | \psi_j \rangle = \delta_{ij}$ gives that the diagonal matrix elements $(A_p)_{i,i}$ are real and that the off-diagonal elements satisfy $(A_p)_{i,j} = (A_p)_{j,i}^*$. The adiabatic parameters are $\lambda_1 = E_R$, $\lambda_2 = E_p$. We calculate the matrix Berry phase for $E_D = E_R - E_0$. The matrix vector potentials with respect to $|\psi\rangle$ and $|\bar{\psi}\rangle$ have the following structures

$$A_{E_R} = \begin{pmatrix} c_1 & a_1 + ib_1 \\ a_1 - ib_1 & -c_1 \end{pmatrix} \\ = a_1 \sigma_x - b_1 \sigma_y + c_1 \sigma_z, \quad (15)$$

and

$$A_{E_p} = \begin{pmatrix} 0 & a_2 - ib_2 \\ a_2 + ib_2 & 0 \end{pmatrix} \\ = a_2 \sigma_x + b_2 \sigma_y. \quad (16)$$

The functions a_i , b_i , and c_i depend on E_R and E_p and are real. Under an adiabatic time evolution a state in this degenerate Hilbert space changes as $|\psi'(t)\rangle = c_1(t)|\psi(t)\rangle + c_2(t)|\bar{\psi}(t)\rangle$. Suppose initially $c_1(0) = 1$ and $c_2(0) = 0$, i.e., $|\psi'(0)\rangle = |\psi\rangle$. The cyclic adiabatic path with the period $T = 2\pi/\omega$ is shown in Fig.1(d) and is given by

$$\begin{pmatrix} E_R(t), E_p(t) \end{pmatrix} = \\ \begin{pmatrix} E_{R,c} + \Delta E_R \cos(\omega t), E_{p,c} + \Delta E_p \sin(\omega t) \end{pmatrix}. \quad (17)$$

The frequency ω can be taken to be a fraction of E_0 . We solve Eq.(2) numerically and find that $c_2(T)$ is non-zero. For the parameters $E_{R,c} = 2E_0$, $E_{p,c} = E_0$, $\Delta E_R = 1.9E_0$, $\Delta E_p = 0.9E_0$, and $\omega = E_0/10$ we find $c_1(T) = 0.8884 - i0.0897$ and $c_2(T) = -0.4429 + i0.0874$.

C. Absence of matrix Berry phase

The matrix Berry phase is absent when the distortion potential $V_p(x, y)$ is zero. When only Rashba and/or Dresselhaus terms are present in Eq.(11) the adiabatic transformation can be performed by varying the parameters $\lambda_1 = E_R$ and $\lambda_2 = E_0$. In this case *an orthonormal basis set exists* in the degenerate Hilbert space such that the matrix vector potentials are diagonal and the non-Abelian Berry phase is zero. This degenerate basis set, $|\psi_1\rangle$ and $|\psi_2\rangle$, has the property that for each k either $c_k^{(1)} = 0$ or $c_k^{(2)} = 0$, where $|\psi_i\rangle = \sum_{k=1}^4 c_k^i |k\rangle$ (see, for example, Eqs.(18) and (19)). The off-diagonal matrix elements of the vector potential $(A_p)_{1,2} = i \langle \psi_1 | \frac{d\psi_2}{d\lambda_p} \rangle = i \sum_{k=1}^4 c_k^{(1)*} \frac{dc_k^{(2)}}{d\lambda_p} = 0$ because either $c_k^{(1)} = 0$ or $c_k^{(2)} = 0$. Therefore the matrix vector potentials are diagonal. In this case the matrix Berry phase will be absent. Let us construct explicitly $|\psi_1\rangle$ and $|\psi_2\rangle$ when only the Rashba term is present, i.e., when $E_D = E_p = 0$. They are given by

$$|\psi_1\rangle = \frac{1}{\sqrt{2}\sqrt{E_0^2 + 9E_R^2 + E_0}\sqrt{E_0^2 + 9E_R^2}} \\ \times \begin{pmatrix} i(E_0 + \sqrt{E_0^2 + 9E_R^2}) \\ 0 \\ 0 \\ 3E_R \end{pmatrix}, \quad (18)$$

and

$$|\psi_2\rangle = \frac{1}{\sqrt{9E_R^2 + (E_0 - \sqrt{E_0^2 + 9E_R^2})^2}} \\ \times \begin{pmatrix} 0 \\ i(E_0 - \sqrt{E_0^2 + 9E_R^2}) \\ 3E_R \\ 0 \end{pmatrix}. \quad (19)$$

With these new eigenstates it is possible to show that the non-Abelian gauge potentials are not only diagonal but also zero: $A_{E_0} = 0$ and $A_{E_R} = 0$.

D. Non-Abelian gauge structure

Let us also test the non-Abelian gauge structure given by Eq. (4). We make a unitary transformation from

$(|\psi_1\rangle, |\psi_2\rangle)$, given in Eqs.(18) and (19), to a pair of time reversed degenerate eigenstates $(|\psi'\rangle, |\bar{\psi}'\rangle)$,

$$|\psi'\rangle = \frac{3}{2\sqrt{E_0^2 + 9E_R^2}} \begin{pmatrix} E_R \\ E_R \\ \frac{1}{3}i \left(E_0 + \sqrt{E_0^2 + 9E_R^2} \right) \\ -\frac{1}{3}i \left(-E_0 + \sqrt{E_0^2 + 9E_R^2} \right) \end{pmatrix}, \quad (20)$$

and

$$|\bar{\psi}'\rangle = \frac{3}{2\sqrt{E_0^2 + 9E_R^2}} \begin{pmatrix} \frac{1}{3}i \left(E_0 + \sqrt{E_0^2 + 9E_R^2} \right) \\ -\frac{1}{3}i \left(-E_0 + \sqrt{E_0^2 + 9E_R^2} \right) \\ E_R \\ E_R \end{pmatrix}. \quad (21)$$

Using these new basis states one can show that the non-Abelian gauge potentials are

$$\begin{aligned} A'_{E_R} &= \frac{1}{E_0^2 + 9E_R^2} \begin{pmatrix} 0 & 3E_0/2 \\ 3E_0/2 & 0 \end{pmatrix} \\ &= \frac{3E_0}{2(E_0^2 + 9E_R^2)} \sigma_x, \end{aligned} \quad (22)$$

and

$$\begin{aligned} A'_{E_0} &= \frac{1}{E_0^2 + 9E_R^2} \begin{pmatrix} 0 & -3E_R/2 \\ -3E_R/2 & 0 \end{pmatrix} \\ &= -\frac{3E_R}{2(E_0^2 + 9E_R^2)} \sigma_x. \end{aligned} \quad (23)$$

Since the old vector potentials are $A_{E_R} = 0$ and $A_{E_0} = 0$ it follows from Eq. (4) that $A'_k = iU \frac{\partial U^\dagger}{\partial \lambda_k}$. We have explicitly verified that this relation holds by computing the unitary matrix U . We have also verified independently the absence of the matrix Berry phase by solving the time-dependent Schrödinger Eq.(2) with A'_{E_R} and A'_{E_0} . This provides a check on our numerical method of solving the time-dependent Schrödinger equation.

IV. DETECTION OF MATRIX BERRY PHASE

After an adiabatic cycle the electron acquires a matrix Berry phase. The presence of such a phase may be detected by measuring the strength of dipole optical transitions before and after the cycle. In this section we calculate the optical strengths using the truncated Hamiltonian. This calculation is not quantitatively accurate but it will give us an estimate of the magnitude of the effect.

First we need to prepare physically some particular pair of degenerate states. In the presence of a magnetic field along the z-axis any degenerate pair of states at zero magnetic field will be split into two states. We define the

lowest energy pair of degenerate eigenstates in the zero magnetic field limit of B_z as $|\psi_1\rangle = \lim_{B_z \rightarrow 0} |\psi_1(B_z)\rangle$ and $|\psi_2\rangle = \lim_{B_z \rightarrow 0} |\psi_2(B_z)\rangle$, where $|\psi_1(B_z)\rangle$ and $|\psi_2(B_z)\rangle$ are the split lowest and next lowest energy states in a finite magnetic field, see Fig.(2a). In a similar way we define degenerate eigenstates in the zero magnetic field limit of B_x : $|\phi_1\rangle = \lim_{B_x \rightarrow 0} |\phi_1(B_x)\rangle$ and $|\phi_2\rangle = \lim_{B_x \rightarrow 0} |\phi_2(B_x)\rangle$, see Fig.(2b). The states $|\psi_{1,2}(B_z)\rangle$ and $|\phi_{1,2}(B_x)\rangle$ are calculated from the Hamiltonian by replacing \vec{k} with $\vec{k} + \frac{e}{c}\vec{A}$ and adding the Zeeman term (\vec{A} is the vector potential and $e > 0$).

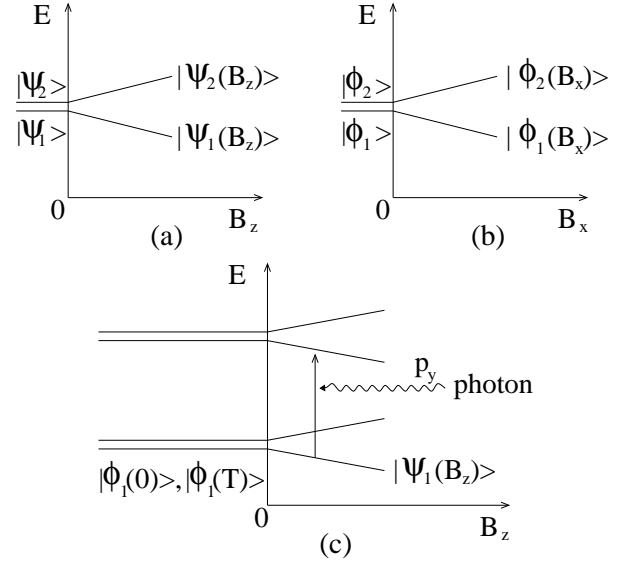


FIG. 2: (a) $|\psi_1\rangle$ and $|\psi_2\rangle$ are generated in the zero limit of B_z . (b) $|\phi_1\rangle$ and $|\phi_2\rangle$ are generated in the zero limit of B_x . (c) Dipole transition from the first to third lowest energy states in a finite B_z . The initial state in this transition is $|\phi_1(0)\rangle$ or $|\phi_1(T)\rangle$. These states represent, respectively, the states before and after the adiabatic cycle. The incident photon is polarized along the y-axis.

Suppose that the electron is in the lowest energy state. In order to measure the matrix Berry phase we perform the following set of procedures:

1. We apply a magnetic field along the x-axis and take the zero field limit. The resulting state $|\phi_1\rangle$ (Fig.(2b)) can be written as a linear combination of $|\psi_1\rangle$ and $|\psi_2\rangle$ (Fig.(2a)): $|\phi_1\rangle = c_1(0)|\psi_1\rangle + c_2(0)|\psi_2\rangle$ with $(|c_1(0)|^2, |c_2(0)|^2) = (1/2, 1/2)$.
2. We apply adiabatically a magnetic field along the z-axis. We increase it to a value B_z . Then the probability for the electron to be in the state $|\psi_1(B_z)\rangle$ is given by $|c_1(0)|^2$.
3. We then measure the intensity of the dipole transition from the lowest energy state to the third lowest energy state, see Fig.(2c). This intensity is proportional to $|c_1(0)|^2$, i.e., proportional to the

probability that $|\psi_1(B_z)\rangle$ is occupied. The lowest and third lowest energy states can be written as a linear combination of the basis states $|mn\sigma\rangle$: both states can be written in the form $c_{0,0,\uparrow}|0,0,\uparrow\rangle + c_{0,1,\uparrow}|0,1,\uparrow\rangle + c_{0,0,\downarrow}|0,0,\downarrow\rangle + c_{0,1,\downarrow}|0,1,\downarrow\rangle$. When the photon is polarized along the y-axis only the basis states with different parities are coupled in the dipole approximation, for example, $\langle 00\sigma|k_y|01\sigma\rangle = -\langle 01\sigma|k_y|00\sigma\rangle$ is non-zero.

Again we assume that the electron is in the lowest energy state. We perform the second set of procedures:

1. We apply a magnetic field along the x-axis and take the zero field limit.
2. We then perform an adiabatic cycle following a closed loop in the parameter space of (E_R, E_p) , given by Eq.(17). After the adiabatic cycle the electron will be in the state $|\phi_1(T)\rangle = c_1(T)|\psi_1\rangle + c_2(T)|\psi_2\rangle$ with $(|c_1(T)|^2, |c_2(T)|^2) = (0.559, 0.441)$ (the parameters are $E_{R,c} = E_0$, $E_{p,c} = 3E_0$, $\Delta E_R = 0.8E_0$, $\Delta E_p = 2.5E_0$, and $\omega = 0.2E_0$).
3. We apply adiabatically a magnetic field along the z-axis, see Fig.(2c). Then the probability for the electron to be in the first lowest energy state, $|\psi_1(B_z)\rangle$, is $|c_1(T)|^2$ while the probability that the electron will be in the second lowest energy state, $|\psi_2(B_z)\rangle$, is $|c_2(T)|^2$. Here the value of B_z is the same as that of step 2 in the first set of procedures.
4. We then measure the intensity of the dipole transition from the lowest energy state to the third lowest energy state. This intensity is proportional to $|c_1(T)|^2$, i.e, proportional to the probability that $|\psi_1(B_z)\rangle$ is occupied.

These two sets of measurements are repeated many times. Then the ratio $\frac{|c_1(T)|^2}{|c_1(0)|^2} = 1.12$ gives the intensity ratio of the dipole transitions in the first and second sets of procedures (the dipole matrix elements cancel each other). It is the direct measure of the matrix Berry phase. The analysis of the intensity of optical transitions in zero magnetic field is complicated due to the presence of the

degeneracy in the final states of the transition. In a finite magnetic field this degeneracy is lifted.

V. DISCUSSIONS

Each discrete eigenstate of a semiconductor quantum dot with the Rashba and/or Dresselhaus spin orbit coupling terms possesses a double degeneracy due to time reversal symmetry. We have investigated the matrix Berry phase of such a quantum dot in a simple truncated model Hamiltonian that can be solved analytically. We have found that the double degeneracy does not necessarily lead to a finite non-Abelian Berry phase. The addition of a parity breaking distortion potential to the Hamiltonian when both the Rashba and Dresselhaus spin orbit coupling terms are present gives rise to a finite matrix Berry phase. We have proposed that this phase may be detected in the dipole transitions between the ground and first excited states in a magnetic field.

For an accurate modeling of possible experiments the number of basis vectors in the truncated Hamiltonian matrix must be chosen sufficiently large. This number will be determined by the ratios $E_0/E_{R,D,p}$. Calculation of the matrix Berry phase in such a case requires a heavy numerical computation. Accurate experimental determination of the functional dependence of the Rashba constant on the electric field would be also valuable in determining the magnitude of matrix Berry phases. The quantum dot studied in this work contains a single electron. It may be worthwhile to investigate the effect of many body interactions.

Acknowledgments

This work was supported by grant No. R01-2005-000-10352-0 from the Basic Research Program of the Korea Science and Engineering Foundation and by Quantum Functional Semiconductor Research Center (QSRC) at Dongguk University of the Korea Science and Engineering Foundation.

-
- [1] D.D. Awschalom, D. Loss, and N. Samarth, *Semiconductor Spintronics and Quantum Computation* (Springer, Berlin, 2002).
 - [2] *Geometric Phases in Physics*, edited by A. Shapere and F. Wilczek (World Scientific, Singapore, 1989).
 - [3] F. Wilczek and A. Zee, *Phys. Rev. Lett.* **52**, 2111 (1984);
 - [4] A. Zee, *Phys. Rev. A* **38**, 1 (1988).
 - [5] J. A. Jones, V. Vedral, A. Ekert and G. Castagnoli, *Nature* **403**, 869 (2000).
 - [6] G. Falci, R. Fazio, G. Palma, J. Siewert and V. Vedral, *Nature* **407**, 355 (2000); M.S. Choi, *J. Phys.: Condens. Matter* **15**, 7823 (2003); L. Faoro, J. Siewert, and R. Fazio, *Phys. Rev. Lett.* **90**, 028301 (2003).
 - [7] J. Pachos and S. Chountasis, *Phys. Rev. A* **62**, 052318 (2000).
 - [8] C. A. Mead, *Phys. Rev. Lett.* **59**, 161 (1987); J. Segert, *J. Math. Phys.* **28**, 2102 (1987).
 - [9] L.-M. Duan, J. I. Cirac and P. Zoller, *Science*, **292** 1965 (2001).
 - [10] R. G. Unanyan, B.W. Shore, and K. Bergmann, *Phys. Rev. A* **59**, 2910 (1999).
 - [11] P. Zanardi and M. Rasetti, *Phys. Lett.* **264**, 94 (1999).
 - [12] J. Pachos, P. Zanardi and M. Rasetti, *Phys. Rev. A* **61**, 010305(R) (2000); J. Pachos and P. Zanardi, *Int. J. Mod.*

- Phys. B **15**, 1257 (2001).
- [13] D. Parodi, M. Sassetti, P. Solinas, P. Zanardi, and N. Zanghi, quant-ph/0510056.
 - [14] E.I. Rashba and A.I. Efros, Phys. Rev. Lett. **91**, 126405 (2003).
 - [15] B. A. Bernevig and S.-C. Zhang, Phys. Rev. B **71**, 035303 (2005).
 - [16] Yu. A. Serebrennikov, Phys. Rev. B **70**, 064422 (2004).
 - [17] P. Solinas, P. Zanardi, N. Zanghi, and F. Rossi, Phys. Rev. A **67**, 062315 (2003).
 - [18] E. Rashba cond-mat/0507007